

Building New IFS Attractors: a Working Vocabulary of Affine Maps

Cédric Bonhomme
cedric@cedricbonhomme.org

July 2026

doi: 10.5281/zenodo.21804778

Abstract

The attractor of an affine iterated function system (IFS) is determined by a handful of real coefficients, yet finding coefficients that produce a *desired* shape remains something of a dark art. We propose a design grammar for the inverse problem: five named families of affine maps (rank-deficient *stem maps*, dominant rotation-contractions for spirals, radial replication for umbrella-shaped inflorescences, golden-angle rotations for phyllotaxis, and non-contracting *symmetry maps*) that compose into new attractors, together with the design loop that turns a family into shipped coefficients. Six original attractors, all distributed in an OCaml library, serve as witnesses. Symmetry maps leave the classical Hutchinson framework and are admissible only under the average contractivity condition of Barnsley, Demko and Elton; we show that this freedom is bounded by a rigidity result, since an attractor whose system contains a scale-one rotation is provably invariant under that rotation. That obstruction explains why a symmetric idealization is the best an IFS can offer for a designed symbol such as the Icelandic vegvísir, and motivates a closing discussion of why grown, organic forms admit compact IFS descriptions while designed ones resist them.

1 Introduction

Iterated function systems are the most economical fractal encoding we know of: four affine maps, twenty-four real numbers, suffice to describe Barnsley’s fern in recognizable botanical detail [BD85, Bar88]. The theory is classical [Hut81, Fal90] and the classical examples are well documented [Vrs, Ses17]. What is documented much less often is the *inverse* craft: given a shape in mind, how does one sit down and write the coefficients? Barnsley’s collage theorem [Bar88] gives the guiding principle (cover the target with small affine copies of itself) but stops short of a design vocabulary.

This note records such a vocabulary, distilled from extending a small OCaml program [Bon26b], started in 2010 during a functional programming course, from eleven classical attractors to sixteen.

Contribution. We claim no new fixed point theory and no new convergence result; the analytical ingredients used here date from 1946 to 1988. The contribution is threefold. First, we synthesize construction techniques that are scattered through the literature and folklore into a reusable vocabulary for *inverse* IFS design: five parametric families, each stated as a formula and witnessed by the exact coefficients of a working attractor (Section 3). Second, we promote the non-contracting symmetry map from a curiosity to a

design device, note that it steps outside the hypotheses of Hutchinson’s theorem, and record the ergodic condition of Elton [Elt87, BE88] under which it remains legitimate, with the verification carried out for our own system. Third, we bound that device with a rigidity result (Proposition 1): an isometry inside the system forces the attractor to be invariant under it, so the cheap route to n -fold symmetry forecloses any asymmetric content. Six original attractors, all shipped in the library, act as witnesses throughout, and Section 6 describes the design loop that produced them.

The author’s earlier report on fractal landscapes [BB08] sits on the opposite side of a divide that this note keeps running into: landscapes call for *statistically* self-similar constructions (midpoint displacement, spectral synthesis), while the plants and symbols considered here are *exactly* self-similar. Section 5 argues that this divide is not an artifact of the tools: forms that grow by repeated local rules are, in a precise sense, IFS attractors already, while forms drawn to carry meaning are not, and the vegvísir experiment of Section 4 makes the argument concrete.

2 Preliminaries

2.1 Hutchinson’s theorem

An *iterated function system* on the plane is a finite family $\{f_1, \dots, f_N\}$ of contractions of $\mathbb{R}^2$; here all maps are affine,

$$f_i(x) = A_i x + b_i, \quad A_i \in \mathbb{R}^{2 \times 2}, \quad b_i \in \mathbb{R}^2,$$

and the contraction factor of f_i is the operator norm $s_i = \|A_i\| < 1$. On the space $\mathcal{H}(\mathbb{R}^2)$ of nonempty compact subsets of $\mathbb{R}^2$, endowed with the Hausdorff metric d_H , the system induces the *Hutchinson operator*

$$W(B) = \bigcup_{i=1}^N f_i(B).$$

$(\mathcal{H}(\mathbb{R}^2), d_H)$ is complete, and W is a contraction with factor $\max_i s_i$; by the Banach fixed point theorem W has a unique fixed point $A = W(A)$, the *attractor*, and $W^n(B) \rightarrow A$ for every seed B [Hut81]. The attractor is exactly self-similar by construction: it is the union of N shrunken affine copies of itself.

When the pieces $f_i(A)$ are essentially disjoint (the open set condition) and the f_i are similarities with ratios s_i , the Hausdorff dimension of A is the unique d solving Moran’s equation $\sum_i s_i^d = 1$ [Mor46, Fal90]: $\ln 3 / \ln 2 \approx 1.585$ for the Sierpiński triangle, $\ln 4 / \ln 3 \approx 1.262$ for the Koch curve. The attractors built in this note overlap by design, so the open set condition fails and Moran’s equation yields only an upper bound; we make no dimension claims for them.

2.2 The chaos game

Rendering $W^n(B)$ directly is exponential in n . The standard alternative is the *chaos game* [Bar88]: attach probabilities $p_i > 0$, $\sum p_i = 1$, to the maps, start from any x_0 , and iterate

$$x_{k+1} = f_{\sigma_k}(x_k), \quad \sigma_k \sim (p_1, \dots, p_N) \text{ i.i.d.},$$

plotting every x_k . The sequence (x_k) almost surely fills the attractor: the empirical measures $\frac{1}{n} \sum_{k < n} \delta_{x_k}$ converge weakly to the unique stationary measure μ of the induced

Markov chain, and $\text{supp } \mu = A$. The probabilities do not change the attractor, only the density with which its parts are visited; a common heuristic [Bar88] takes p_i proportional to $|\det A_i|$, the area contraction of f_i , so that all parts of the image darken at a comparable rate.

Crucially for Section 3.5, the chaos game converges under a condition strictly weaker than contractivity of every map. It suffices that the system be *contracting on average* [Elt87, BE88]:

$$\sum_{i=1}^N p_i \ln s_i < 0, \quad (1)$$

i.e. maps with $s_i \geq 1$ are permitted as long as the contracting ones carry enough probability. Under (1) the chain still has a unique attracting stationary measure μ , and we simply *define* the attractor as $\text{supp } \mu$; since $\mu = \sum_i p_i \mu \circ f_i^{-1}$ and all $p_i > 0$, this support again satisfies the self-similarity equation $A = \bigcup_i f_i(A)$.

2.3 Notation and implementation

Throughout, coefficients are written in the convention of the accompanying library [Bon26b]: a map is the array $[a; b; c; d; e; f]$ meaning

$$x' = ax + by + c, \quad y' = dx + ey + f,$$

and a similarity of ratio s and angle θ centred at the origin is $[s \cos \theta; -s \sin \theta; t_x; s \sin \theta; s \cos \theta; t_y]$. The library stores an attractor as a viewport plus a list of maps with cumulative probability thresholds:

```
type point = { x : float; y : float }
type transfo = { pb : float; kf : float array }
type ifs = { po : point; sz : point; lt : transfo list }
```

and the chaos game itself is a dozen lines of OCaml. All figures in this note are drawn by that code (or by a line-for-line Python port used during design); every coefficient shown is the one shipped in the library.

3 A working vocabulary of maps

Designing an attractor is choosing which affine copies of the *whole* image make up the image. In practice a small number of map families cover a surprising amount of ground. We name each family, give its parametric form, and exhibit it in a shipped attractor.

3.1 Stem maps: rank-deficient terminators

The oldest trick in the book is a map whose matrix is (near) rank one:

$$T = \begin{pmatrix} \varepsilon & 0 \\ 0 & \lambda \end{pmatrix}, \quad \varepsilon \approx 0, \lambda < 1,$$

which collapses the entire attractor onto a thin vertical segment: a *stem*. Vrscaj's lecture notes [Vrs] aptly call these "terminators". Barnsley's fern uses the exact rank-one case $[0; 0; 0; 0.16; 0]$ for its stem; the tree attractors in [Vrs] use $\varepsilon = 0$ with $\lambda \approx 0.5$.

Two of our attractors extend the idiom. The stem of the wild carrot (Section 3.3) is $[0.02; 0; 0; 0; 0.55; 0]$: not exactly rank one, so the "stem" is a very thin copy of the whole

plant, which gives it a natural, slightly textured silhouette for free. The arm shaft of the vegvisir (Section 4) is the horizontal analogue $[0.40; 0; 0.45; 0; 0.04; 0]$: a long flat copy of the whole symbol, whose own eightfold detail decorates the shaft with rune-like marks, again for free. This is the general lesson of stem maps: because the squashed object is the whole attractor rather than a line segment, its residual thickness carries self-similar ornament at no extra cost in coefficients.

3.2 Spiral generators: one dominant rotation-contraction

A single similarity $sR(\theta)$ with s close to 1, applied with high probability, drives every orbit along the logarithmic spiral $r = s^t$, $\varphi = t\theta$. Satellite maps then plant a motif on the spiral, and since the motif is a copy of the whole attractor, the result is a spiral of spirals.

Our **fiddlehead** (Figure 1, left), named after the coiled tip of a young fern frond, is the three-map instance

map	ratio s	angle θ	translation	p_i
main curl	0.92	$+20^\circ$	$(0, 0.40)$	0.85
offshoot	0.30	-60°	$(0.60, 0.40)$	0.10
seed	0.15	0°	$(0, 0)$	0.05

The dominant map carries the mass around the curl; the offshoot plants a shrunken, counter-rotated copy of the whole attractor at regular intervals along it. The same architecture, with different parameters, underlies the classical **galaxy** attractor that has been in the library since 2010: a case of convergent evolution that illustrates how small this design space really is. The maple leaf (Figure 1, right) is the classical four-map system in the same spirit, included in the library for comparison.

Three parameters govern the visual result, and each fails in a characteristic way. The ratio s sets how many turns are visible before the curl vanishes into the fixed point: at $s = 0.92$ and $\theta = 20^\circ$ the orbit loses a factor $0.92^{18} \approx 0.22$ per full turn, giving the two-and-a-half visible whorls of Figure 1; pushing s to 0.98 crowds the turns into an indistinct disk, dropping it to 0.8 leaves barely one. The angle θ sets the pitch, and must not divide 360° evenly, or the offshoots stack onto a few rays instead of sweeping the spiral. The offshoot's own rotation matters most: a positively rotated offshoot spirals in the same direction as its parent and the eye reads one smooth curve, whereas the counter-rotation used here makes each offshoot legibly a *separate* smaller curl, which is what produces the spiral-of-spirals impression. No other family in this vocabulary reproduces that effect. Radial replication (Section 3.3) would place copies around a circle with no accumulation toward a centre, and a symmetry map (Section 3.5) would impose exact discrete symmetry, which is precisely what a spiral must not have.

3.3 Radial replication: umbels

The flower of the wild carrot (*Daucus carota*, Queen Anne's lace) is an *umbel*: stalks radiate from one point to a domed rim, each stalk ending in a smaller umbel, and so on for two or three botanical levels; the attractor continues the recursion forever. The construction places k shrunken, slightly rotated copies of the whole plant along the rim of a dome, plus one stem map:

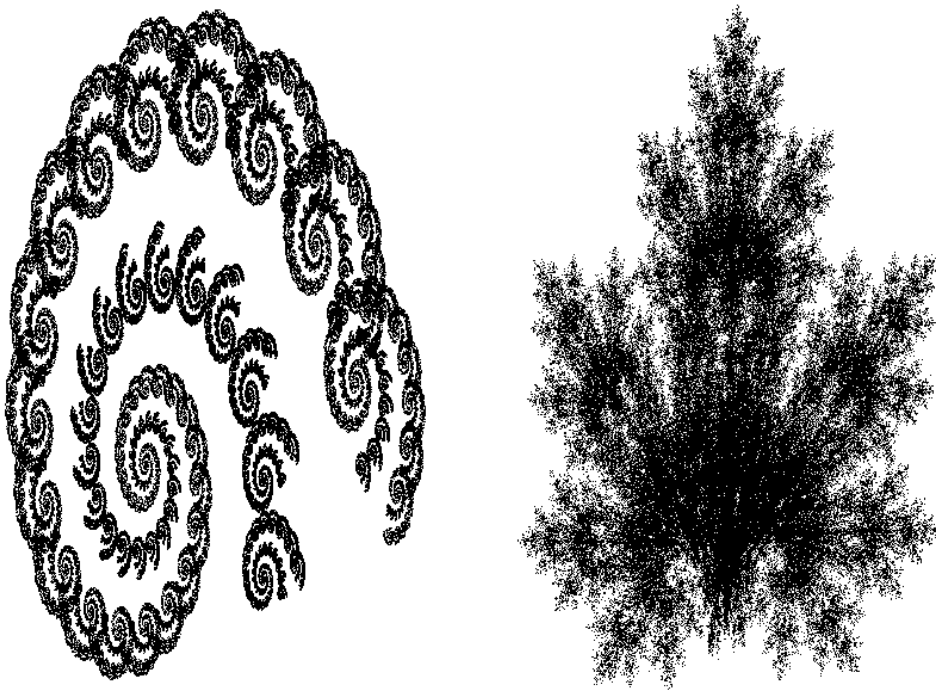


Figure 1: Left: `fiddlehead`, a spiral of spirals from three maps (Section 3.2). Right: `maple`, a classical four-map leaf. Chaos game, 2×10^5 points each.

map	ratio s	angle θ	translation	p_i
rim, far left	0.30	-28°	$(-0.70, 0.38)$	0.18
rim, left	0.36	-14°	$(-0.43, 0.50)$	0.18
rim, top	0.38	0°	$(0, 0.55)$	0.18
rim, right	0.36	$+14^\circ$	$(0.43, 0.50)$	0.18
rim, far right	0.30	$+28^\circ$	$(0.70, 0.38)$	0.18
stem	$[0.02; 0; 0; 0; 0.55; 0]$		$(0, 0)$	0.10

The rotations lean the outer copies outward, following the dome’s normal directions, and the decreasing ratios toward the edge flatten the silhouette exactly as gravity does for the plant. Thirty-six numbers in total (Figure 2, right).

Two design decisions carry the image. The first is that the ratios decrease away from the centre (0.38 at the top, 0.30 at the rim) while the rotations increase (0° to $\pm 28^\circ$): equal ratios and no rotation would produce a flat row of identical tufts, which reads as a diagram rather than a flower, since real umbels are domes whose peripheral rays are both shorter and tilted. The second is the choice of an odd number of rim maps. With five, one map sits on the axis of symmetry and the two flanking pairs mirror it, so the attractor inherits a single reflection axis, the bilateral symmetry of a plant standing in gravity. A symmetry map would have been the cheaper way to obtain regularity here, and it is exactly the wrong tool: Proposition 1 would force full C_k invariance, turning the umbel into a rosette with no distinguished vertical and no place to attach a stem.

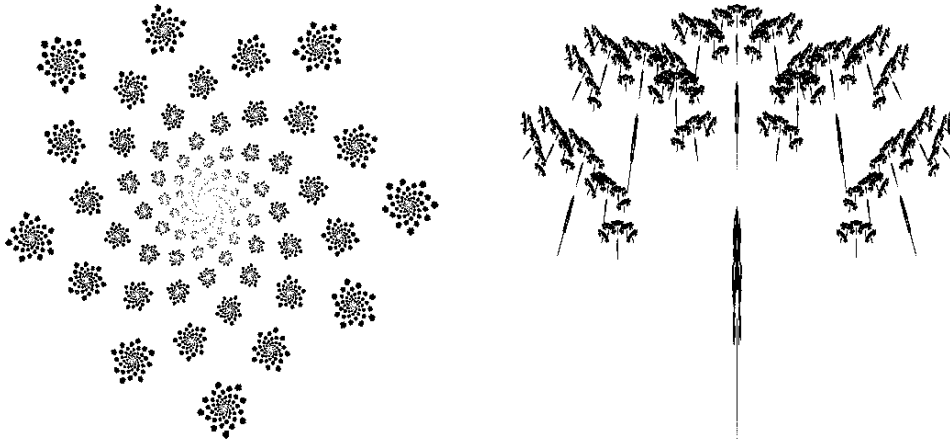


Figure 2: Left: **sunflower**, golden-angle phyllotaxis from two maps (Section 3.4), 5×10^5 points. Right: **lace**, an umbel of umbels from six maps (Section 3.3), 3×10^5 points.

3.4 Golden-angle maps: phyllotaxis in two transforms

Sunflower heads, daisy capitula and romanesco spirals place successive primordia at the *golden angle* $\alpha = 360^\circ(1 - 1/\varphi) \approx 137.508^\circ$ from one another, where φ is the golden ratio [Vog79, PL90]. Because φ has continued fraction $[1; 1, 1, \dots]$, it is the number worst approximated by rationals, so the angles $k\alpha \bmod 360^\circ$ never lock onto spokes and fill the circle as evenly as possible; this is the equidistribution that makes real seed heads look uniformly packed.

The IFS rendition needs exactly two maps:

map	ratio s	angle θ	translation	p_i
whorl	0.97	137.508°	(0, 0)	0.88
bud	0.13	0°	(1, 0)	0.12

The bud map plants a small copy of the whole attractor at the rim point (1, 0); the whorl map sends bud k to bud $k + 1$, rotated by α and pulled inward by 0.97. The k -th bud therefore sits at angle $k\alpha$ and radius 0.97^k , and since every bud is a copy of the whole attractor, each is itself a rosette of rosettes (Figure 2, left).

One honest caveat: real capitula follow Vogel's spiral $r = c\sqrt{k}$ [Vog79], which keeps the areal density of organs constant, and the computer graphics literature has since generalized it to arbitrary surfaces of revolution [Rid86, PFKM03]. An affine IFS can only produce the geometric law $r = s^k$, since each application of the whorl map multiplies the radius by a constant. Our attractor is thus a *logarithmic* phyllotaxis, denser toward the rim than a true sunflower; the golden-angle equidistribution, which dominates the visual impression, is nevertheless exact. It is worth noting that the golden angle itself is empirical input in both models: it is reproduced, not explained [PFKM03].

3.5 Symmetry maps: non-contracting members

All previous maps were contractions. The last family is not: a *pure rotation at scale one*,

$$R = R(\beta), \quad s_R = 1,$$

included in the system with substantial probability. R draws nothing new by itself; in the chaos game it teleports the current point around the attractor, replicating whatever the other maps produce into every rotated position. It is a distribution-of-labour device: the contracting maps need only describe *one* sector of the figure, and R propagates that sector around the circle.

Hutchinson’s theorem no longer applies, since W is not a contraction on $(\mathcal{H}, d_H)$. But the average contractivity condition (1) is easily satisfied: in the vegvísir system below, the rotation carries probability 0.58 and the remaining maps have ratios between 0.10 and 0.40, giving $\sum_i p_i \ln s_i \approx -0.66 < 0$. Elton’s theorem [Elt87] then guarantees a unique attracting invariant measure, and the attractor, its support, still satisfies $A = \bigcup_i f_i(A)$.

The price is paid in rendering: since more than half of all chaos game steps apply R , which revisits existing structure rather than refining it, the attractor needs noticeably more iterations to fill in (5×10^5 where the fern needs 2×10^5). The probability 0.58 was tuned empirically: too low and the arms far from the generating sector stay faint, too high and the contracting maps starve.

What a symmetry map can never do is break its own symmetry:

Proposition 1. *Let $\{f_1, \dots, f_N\}$ with probabilities satisfying (1) contain a map R that is a surjective isometry of $\mathbb{R}^2$ (e.g. a rotation), and let $A = \text{supp } \mu$ be the attractor. Then $R(A) = A$.*

Proof. From $A = \bigcup_i f_i(A) \supseteq R(A)$, the isometry R maps the compact set A into itself. An isometry of a compact metric space into itself is surjective: if some $x \in A$ had $d(x, R(A)) = \varepsilon > 0$, the orbit $x_n = R^n(x)$ would satisfy $d(x_n, x_m) = d(x, R^{m-n}(x)) \geq \varepsilon$ for all $m > n$, since $R^{m-n}(x) \in R(A)$; a sequence in a compact set with no convergent subsequence is impossible. Hence $R(A) = A$. $\square$

With $\beta = 45^\circ$ the attractor is therefore invariant under the cyclic group C_8 : *exactly* eightfold symmetric, whatever the other maps do. This is both the power of the technique and, as the next section shows, its limit.

4 Case study: a designed symbol

The vegvísir is an Icelandic magical stave, a “wayfinder”, known from nineteenth-century manuscripts: eight arms radiate from a centre, and each arm ends in a *different* rune-like device. It is a natural challenge for the vocabulary above: can an IFS draw it?

Strictly, no, and the obstruction is Proposition 1 read in reverse. The economical way to draw eight arms is a 45° symmetry map, but then the attractor is exactly C_8 -symmetric and all arms are congruent: distinct terminal runes are ruled out. Renouncing the symmetry map does not help at reasonable cost: each of the eight distinct rune motifs would need its own group of maps, none of which may be an image of the whole (the attractor equation $A = \bigcup f_i(A)$ forces every map’s output to be a copy of the entire symbol), so one is pushed toward IFS with condensation or simply many dozens of coefficients, forfeiting the economy that makes the exercise interesting. The deeper reason is aesthetic rather than technical: the vegvísir was drawn by hand to *mean* something, its asymmetry is the content, and exact self-similarity has no room for it.

What an IFS can offer is a symmetric idealization (Figure 3): an eight-armed rune star, every arm ending in an infinitely recursive miniature of the whole. The system is six maps:

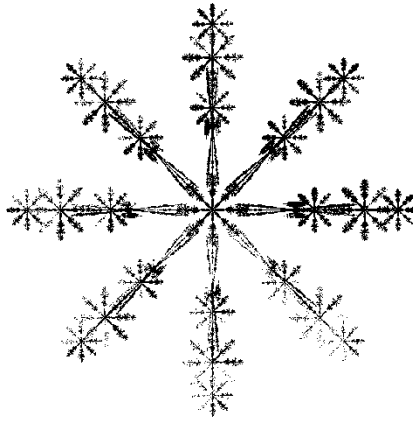


Figure 3: The **vegvisir** attractor: six maps, C_8 -symmetric by Proposition 1. Chaos game, 5×10^5 points.

map	coefficients $[a; b; c; d; e; f]$	role	p_i
symmetry	$[0.7071; -0.7071; 0; 0.7071; 0.7071; 0]$	$R(45^\circ)$, $s = 1$	0.58
shaft	$[0.40; 0; 0.45; 0; 0.04; 0]$	stem map, horizontal	0.18
crossbar 1	$[0; -0.11; 0.55; 0.11; 0; 0]$	$0.11 R(90^\circ)$ at $x=0.55$	0.08
crossbar 2	$[0; -0.15; 0.82; 0.15; 0; 0]$	$0.15 R(90^\circ)$ at $x=0.82$	0.08
tip	$[0.10; 0; 1.00; 0; 0.10; 0]$	miniature at arm's end	0.05
centre	$[0.18; 0; 0; 0; 0.18; 0]$	miniature at origin	0.03

The five contracting maps describe a single horizontal arm: a flat shaft (Section 3.1), two rotated crossbar copies whose own eightfold structure decorates them with rune-like detail, a tip miniature, and a centre filler. The symmetry map replicates the arm eight times. The result is not a *vegvisir*; it is what the *vegvisir* would be if it were grown by a rule instead of drawn by a hand, and the family resemblance is exactly the part of the symbol that is rule-like.

5 Why nature cooperates

Half of the attractors above depict plants, and they are by a wide margin the most convincing ones. The library's history repeats the observation: of its sixteen attractors, the memorable ones are the fern, the maple leaf, the tree, the fiddlehead, the sunflower, the wild carrot; the geometric and symbolic ones read as diagrams. The explanation is not that plants are pretty, but that the encoding fits.

A plant does not execute a blueprint of its final shape. It grows by repeating local rules: sprout, shrink, turn, branch. Its adult form is the accumulation of one rule applied at every scale, which is precisely the fixed point equation $A = \bigcup_i f_i(A)$. Developmental biology formalizes the same observation as L-systems, parallel rewriting systems in which each module of the organism is advanced by rules depending only on its own state and its neighbours [PL90]; in the vocabulary of [PFKM03], a growing plant is a *structured dynamical system* whose transition function is local. Locality plus repetition is self-similarity. The fern is not *like* a fractal; within the resolution at which physiology cuts off the recursion, it *is* the attractor of a handful of affine maps, which is why Barnsley could capture it in

twenty-four coefficients [BD85]. The romanesco that illustrated self-similarity in the author’s 2008 report [BB08] makes the same point in three dimensions, and its golden-angle spirals are the ones distilled into the two-map system of Section 3.4.

The comparison also marks the boundary of what an IFS can say. An L-system models development *in time*, and its later refinements add exactly the ingredients an IFS lacks: context sensitivity, positional information along an axis, competition for light and space, and biomechanics [PFKM03]. An IFS keeps only the limit of that process, and only when the process is context-free and strictly self-similar. Our `lace` is an idealized umbel whose every ray is a copy of the whole, whereas a real *Daucus carota* has rays that shorten toward the rim under positional control; the attractor buys its thirty-six coefficients by discarding that regulation. The right reading is not that plants are IFS, but that the deterministic, context-free core of plant development is exactly the part an IFS can capture, and that this core already accounts for most of what makes a fern look like a fern.

Designed artifacts sit at the other pole. A symbol like the *vegvísir* is drawn once, at one scale, to carry meaning; nothing in its production iterates, and Section 4 showed how the mathematics enforces the mismatch. Between the two poles lie nature’s *statistically* self-similar forms: coastlines, clouds, mountain relief. These grow by iteration too, but by noisy iteration, and their proper tools are the random constructions (midpoint displacement, fractional Brownian surfaces) surveyed in [Man82, BB08]; a deterministic IFS renders them as too regular, just as it renders the *vegvísir* as too symmetric. The rule of thumb the vocabulary suggests is simple: exact self-similarity for what grows by a deterministic rule, randomness for what grows by a noisy one, and resignation for what does not grow at all.

6 Implementation notes

Everything above is implemented in `ifs-fractals` [Bon26b], an OCaml library and command-line tool (about two hundred lines in total, chaos game included) released under the GPL and distributed through opam:

```
$ opam install ifs-fractals
$ ifs-fractals lace
```

The design loop deserves a methodological remark, since it is what makes the vocabulary usable in practice. Coefficients were never written blind: each candidate system was first simulated (a Python port of the chaos game, a few dozen lines), rendered to an image, inspected, and iterated upon; the simulation also computed the attractor’s bounding box, which becomes the viewport stored with the system. The loop from a parametric family of Section 3 to shipped coefficients took minutes per attractor. Readers wishing to extend the collection will find that the vocabulary plus this simulate-inspect loop makes the inverse problem, if not solved, at least workmanlike.

Use of a language model

The six new attractors were designed in 2026 in an interactive session with Claude Fable 5, a large language model by Anthropic, and it would be misleading to describe the division of labour vaguely. The author set the goals and constraints, ran and inspected the renderings, decided which candidates to keep, and checked the statements made here; the model proposed the botanical subjects, derived and tuned the coefficient sets inside the simulate-inspect loop above, and drafted the exposition, including the proof of Propo-

sition 1. Responsibility for the correctness of the mathematics and for the final content rests with the author; the model is a tool, not an author.

Two of its behaviours are worth recording for readers considering the same workflow. Asked for the *vegvísir*, it identified the obstruction of Section 4 *before* attempting a construction, rather than returning a plausible but wrong system; and asked whether *fiddlehead* was original, it traced the architecture back to the *galaxy* attractor already present in the library, which is the provenance reported in Section 3.2. A narrative account of the sessions is given in [Bon26a].

7 Conclusion

Affine IFS design is usually presented as a gallery of received coefficient tables. Naming the construction techniques (stem maps, spiral generators, radial replication, golden-angle maps, symmetry maps) turns the gallery into a grammar: the six attractors added to a sixteen-year-old library in 2026 were all obtained by composing these families and tuning by inspection, in minutes each. The grammar also knows its own limits, and states one of them precisely: Proposition 1 says that the cheap way to eightfold symmetry forecloses any asymmetric content, which is why grown forms fit in a few dozen coefficients while a hand-drawn symbol never will. The gap between a fern and a *vegvísir* is not twenty-four coefficients; it is the difference between form produced by iteration and form produced by intention.

References

- [Bar88] Michael F. Barnsley. *Fractals Everywhere*. Academic Press, Boston, 1988.
- [BB08] Cédric Bonhomme and Jean-Charles Bettinger. Paysages fractals. Technical report, Université Paul Verlaine – Metz, UFR MIM, 2008. Supervised by Dominique Michel.
- [BD85] Michael F. Barnsley and Stephen Demko. Iterated function systems and the global construction of fractals. *Proceedings of the Royal Society of London A*, 399:243–275, 1985.
- [BE88] Michael F. Barnsley and John H. Elton. A new class of Markov processes for image encoding. *Advances in Applied Probability*, 20(1):14–32, 1988.
- [Bon26a] Cédric Bonhomme. Challenging Claude’s creativity with IFS fractals and OCaml. Blog post, <https://www.cedricbonhomme.org/2026/07/27/challenging-claude-with-ifs-fractals/>, 2026.
- [Bon26b] Cédric Bonhomme. ifs-fractals: IFS fractals drawn with the chaos game in OCaml. opam package, <https://github.com/cedricbonhomme/iterated-function-systems>, 2026.
- [Elt87] John H. Elton. An ergodic theorem for iterated maps. *Ergodic Theory and Dynamical Systems*, 7(4):481–488, 1987.
- [Fal90] Kenneth Falconer. *Fractal Geometry: Mathematical Foundations and Applications*. John Wiley & Sons, Chichester, 1990.

- [Hut81] John E. Hutchinson. Fractals and self-similarity. *Indiana University Mathematics Journal*, 30(5):713–747, 1981.
- [Man82] Benoît B. Mandelbrot. *The Fractal Geometry of Nature*. W. H. Freeman, San Francisco, 1982.
- [Mor46] Patrick A. P. Moran. Additive functions of intervals and Hausdorff measure. *Proceedings of the Cambridge Philosophical Society*, 42(1):15–23, 1946.
- [PFKM03] Przemysław Prusinkiewicz, Pavol Federl, Radosław Karwowski, and Radomír Měch. L-systems and beyond. SIGGRAPH 2003 Course Notes, 2003.
- [PL90] Przemysław Prusinkiewicz and Aristid Lindenmayer. *The Algorithmic Beauty of Plants*. Springer-Verlag, New York, 1990.
- [Rid86] J. N. Ridley. Ideal phyllotaxis on general surfaces of revolution. *Mathematical Biosciences*, 79(1):1–24, 1986.
- [Ses17] Olivier Sester. Les systèmes de fonctions itérées. Workshop slides, Université Paris-Est Marne-la-Vallée, 2017.
- [Vog79] Helmut Vogel. A better way to construct the sunflower head. *Mathematical Biosciences*, 44(3–4):179–189, 1979.
- [Vrs] Edward R. Vrscay. Iterated function systems and the construction of fractal sets. Lecture notes (Lectures 33–35), Department of Applied Mathematics, University of Waterloo.